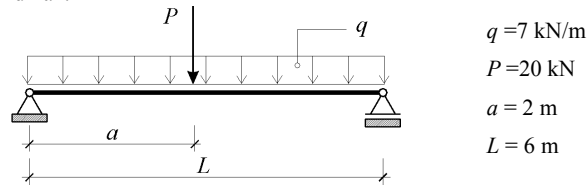


## 5. Zadatak

### 5a.

Dimenzionisati nosač sistema proste grede koji je opterećen prema skici. Nosač je kontinualno bočno pridržan.



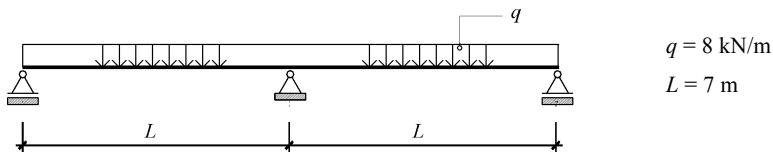
Poprečni presek nosača usvojiti iz asortimana vruće valjanih profila:

IPE	HEB	HEM
Osnovni materijal:	Č0361	
Slučaj opterećenja:	II	
Dozvoljeni ugib nosača:	L/250	

### 5b.

Dimenzionisati nosač sistema kontinualne grede na dva polja koji je opterećen jednakopodeljenim opterećenjem. Gornja nožica nosača je bočno pridržana:

- nad osloncima i u sredinama raspona,



Poprečni presek nosača usvojiti iz asortimana vruće valjanih profila:

IPE	
Osnovni materijal:	Č0361
Slučaj opterećenja:	I
Dozvoljeni ugib nosača:	L/300

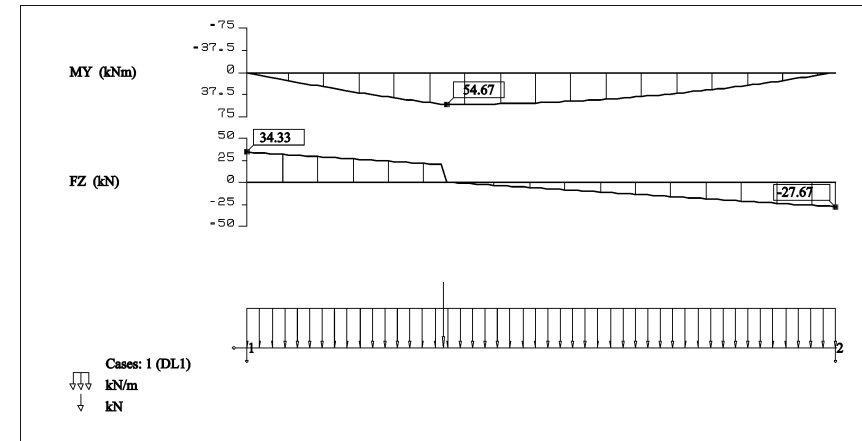
### 5a)

Osnovni materijal Č0361 (II sl.o.)  $\Rightarrow$  dopušten normalni napon  $\sigma_{dop} = 18 \text{ kN/cm}^2$

dopušten smičući napon  $\tau_{dop} = 10 \text{ kN/cm}^2$

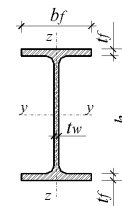
### Dimenzionisanje nosača:

Uticaji u nosaču



$$W_{pot} = \frac{M_{max}}{\sigma_{dop}} = \frac{54,67}{18} = 303,7 \text{ cm}^3$$

### a) Poprečni presek IPE 240

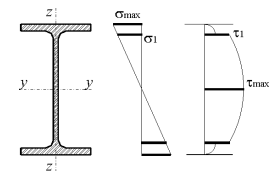


$b_f = 120 \text{ mm}$	$A = 39,1 \text{ cm}^2$	$W_y = 324 \text{ cm}^3$
$t_f = 9,8 \text{ mm}$	$I_y = 3890 \text{ cm}^4$	$W_z = 47,3 \text{ cm}^3$
$h = 240 \text{ mm}$	$I_z = 284 \text{ cm}^4$	$S_y = 183 \text{ cm}^3$
$t_w = 6,2 \text{ mm}$	$i_y = 9,97 \text{ cm}$	$G = 30,7 \text{ kg/m}$
	$i_z = 2,69 \text{ cm}$	

Kontrola napona:

$$\sigma_{max} = \frac{M_{max}}{W_y} = \frac{5467}{324} = 16,87 \text{ kN/cm}^2 < \sigma_{dop} = 18 \text{ kN/cm}^2$$

$$\tau_{max} = \frac{T_{max} \cdot S_y}{t_w \cdot I_y} = \frac{34,33 \cdot 183}{0,62 \cdot 3890} = 2,6 \text{ kN/cm}^2 < \tau_{dop} = 10 \text{ kN/cm}^2$$



Kontrola uporednog napona na mestu maksimalnog momenta savijanja

$$S_{y,0} = (12 \cdot 0,98) \cdot \frac{24 - 0,98}{2} = 135,4 \text{ cm}^3$$

$$T_{odg} = 34,33 - 2 \cdot 7 = 20,33 \text{ kN}$$

$$s_1 = \frac{M_{\max}}{I_y} \cdot \frac{h - 2t_f}{2} = \frac{5467}{3890} \cdot \frac{(24 - 2 \cdot 0,98)}{2} = 15,5 \text{ kN/cm}^2 < s_{dop} = 18 \text{ kN/cm}^2$$

$$t_1 = \frac{T \cdot S_{y,0}}{t_w \cdot I_y} = \frac{20,33 \cdot 135,4}{0,62 \cdot 3890} = 1,14 \text{ kN/cm}^2 < t_{dop} = 10 \text{ kN/cm}^2$$

$$s_u = \sqrt{s_1^2 + 3 \cdot t_1^2} = \sqrt{15,5^2 + 3 \cdot 1,14^2} = 15,6 \text{ kN/cm}^2 < s_{dop} = 18 \text{ kN/cm}^2$$

Kontrola ugiba:

$$f_{\max}(q) = \frac{5 \cdot q \cdot l^4}{384 \cdot I_y \cdot E} = \frac{5 \cdot 0,07 \cdot 600^4}{384 \cdot 21000 \cdot 3890} = 1,4 \text{ cm}$$

$$f_{\max}(P) = \frac{P}{27 \cdot E \cdot I_y} \cdot \frac{a}{l} \cdot \sqrt{3 \cdot (l^2 - a^2)^3} = \frac{20}{27 \cdot 21000 \cdot 3890} \cdot \frac{200}{600} \cdot \sqrt{3 \cdot (600^2 - 200^2)^3}$$

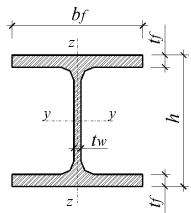
$$f_{\max}(P) = 0,95 \text{ cm}$$

Položaj maksimalnog ugiba usled sile P = 20 kN

$$x = \sqrt{\frac{l^2 - a^2}{3}} = \sqrt{\frac{600^2 - 200^2}{3}} = 326 \text{ cm} \approx 300 \text{ cm} = \frac{l}{2}$$

$$f_{\max}(q+P) = 1,4 + 0,95 = 2,35 \text{ cm} < 2,4 \text{ cm} = l/250 = f_{dop}$$

## b) Poprečni presek HEB 180



$b_f=180 \text{ mm}$	$A=65,3 \text{ cm}^2$	$W_y=426 \text{ cm}^3$
$t_f=1,4 \text{ mm}$	$I_y=3830 \text{ cm}^4$	$W_z=151 \text{ cm}^3$
$h=180 \text{ mm}$	$I_z=1360 \text{ cm}^4$	$S_y=241 \text{ cm}^3$
$t_w=8,5 \text{ mm}$	$i_y=7,66 \text{ cm}$	$G=51,2 \text{ kg/m}$
	$i_z=4,57 \text{ cm}$	

Kontrola napona:

$$s_{\max} = \frac{M_{\max}}{W_y} = \frac{5467}{426} = 12,8 \text{ kN/cm}^2 < s_{dop} = 18 \text{ kN/cm}^2$$

$$t_{\max} = \frac{T_{\max} \cdot S_y}{t_w \cdot I_y} = \frac{34,33 \cdot 241}{0,85 \cdot 3830} = 2,54 \text{ kN/cm}^2 < t_{dop} = 10 \text{ kN/cm}^2$$

Kontrola uporednog napona na mestu maksimalnog momenta savijanja

$$S_{y,0} = (18 \cdot 1,4) \cdot \frac{18 - 1,4}{2} = 209,2 \text{ cm}^3$$

$$T_{odg} = 34,33 - 2 \cdot 7 = 20,33 \text{ kN}$$

$$s_1 = \frac{M_{\max}}{I_y} \cdot \frac{h - 2t_f}{2} = \frac{5467}{3830} \cdot \frac{(18 - 2 \cdot 1,4)}{2} = 10,85 \text{ kN/cm}^2 < s_{dop} = 18 \text{ kN/cm}^2$$

$$t_1 = \frac{T \cdot S_{y,0}}{t_w \cdot I_y} = \frac{20,33 \cdot 209,2}{0,85 \cdot 3830} = 1,31 \text{ kN/cm}^2 < t_{dop} = 10 \text{ kN/cm}^2$$

$$s_u = \sqrt{s_1^2 + 3 \cdot t_1^2} = \sqrt{10,85^2 + 3 \cdot 1,31^2} = 11,1 \text{ kN/cm}^2 < s_{dop} = 18 \text{ kN/cm}^2$$

Kontrola ugiba:

$$f_{\max}(q) = \frac{5 \cdot q \cdot l^4}{384 \cdot I_y \cdot E} = \frac{5 \cdot 0,07 \cdot 600^4}{384 \cdot 21000 \cdot 3830} = 1,47 \text{ cm}$$

$$f_{\max}(P) = \frac{P}{27 \cdot E \cdot I_y} \cdot \frac{a}{l} \cdot \sqrt{3 \cdot (l^2 - a^2)^3} = \frac{20}{27 \cdot 21000 \cdot 3830} \cdot \frac{200}{600} \cdot \sqrt{3 \cdot (600^2 - 200^2)^3}$$

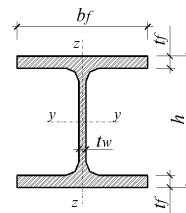
$$f_{\max}(P) = 0,96 \text{ cm}$$

Položaj maksimalnog ugiba usled sile P = 20 kN

$$x = \sqrt{\frac{l^2 - a^2}{3}} = \sqrt{\frac{600^2 - 200^2}{3}} = 326 \text{ cm} \approx 300 \text{ cm} = \frac{l}{2}$$

$$f_{\max}(q+P) = 1,47 + 0,96 = 2,43 \text{ cm} \approx 2,4 \text{ cm} = l/250 = f_{dop}$$

## c) Poprečni presek HEM 160



$b_f=166 \text{ mm}$	$A=97,1 \text{ cm}^2$	$W_y=566 \text{ cm}^3$
$t_f=23 \text{ mm}$	$I_y=5100 \text{ cm}^4$	$W_z=212 \text{ cm}^3$
$h=180 \text{ mm}$	$I_z=1760 \text{ cm}^4$	$S_y=337 \text{ cm}^3$
$t_w=14,5 \text{ mm}$	$i_y=7,25 \text{ cm}$	$G=76,2 \text{ kg/m}$
	$i_z=4,26 \text{ cm}$	

Kontrola napona:

$$s_{\max} = \frac{M_{\max}}{W_y} = \frac{5467}{566} = 9,7 \text{ kN/cm}^2 < s_{dop} = 18 \text{ kN/cm}^2$$

$$t_{\max} = \frac{T_{\max} \cdot S_y}{t_w \cdot I_y} = \frac{34,33 \cdot 337}{1,45 \cdot 5100} = 1,56 \text{ kN/cm}^2 < t_{dop} = 10 \text{ kN/cm}^2$$

Kontrola uporednog napona na mestu maksimalnog momenta savijanja

$$S_{y,0} = (16,6 \cdot 2,3) \cdot \frac{18 - 2,3}{2} = 299,7 \text{ cm}^3$$

$$T_{odg} = 34,33 - 2 \cdot 7 = 20,33 \text{ kN}$$

$$s_1 = \frac{M_{\max}}{I_y} \cdot \frac{h - 2t_f}{2} = \frac{5467}{5100} \cdot \frac{(18 - 2 \cdot 2,3)}{2} = 7,2 \text{ kN/cm}^2 < s_{dop} = 18 \text{ kN/cm}^2$$

$$t_1 = \frac{T \cdot S_{y,0}}{t_w \cdot I_y} = \frac{20,33 \cdot 299,7}{1,45 \cdot 5100} = 0,82 \text{ kN/cm}^2 < t_{dop} = 10 \text{ kN/cm}^2$$

$$s_u = \sqrt{s_1^2 + 3 \cdot t_1^2} = \sqrt{7,2^2 + 3 \cdot 0,82^2} = 7,33 \text{ kN/cm}^2 < s_{dop} = 18 \text{ kN/cm}^2$$

Kontrola ugiba:

$$f_{max}(q) = \frac{5 \cdot q \cdot l^4}{384 \cdot I_y \cdot E} = \frac{5 \cdot 0,07 \cdot 600^4}{384 \cdot 21000 \cdot 5100} = 1,1 \text{ cm}$$

$$f_{max}(P) = \frac{P}{27 \cdot E \cdot I_y} \cdot \frac{a}{l} \cdot \sqrt{3 \cdot (l^2 - a^2)^3} = \frac{20}{27 \cdot 21000 \cdot 5100} \cdot \frac{200}{600} \cdot \sqrt{3 \cdot (600^2 - 200^2)^3}$$

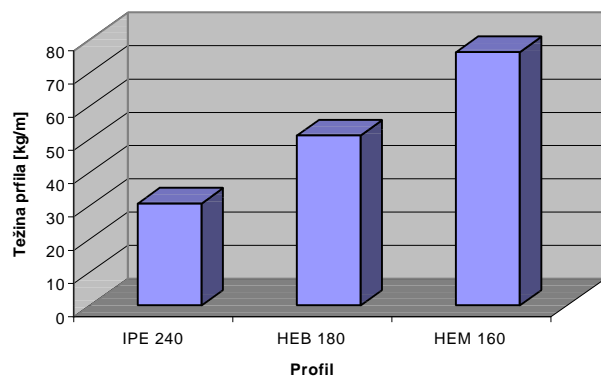
$$f_{max}(P) = 0,72 \text{ cm}$$

Položaj maksimalnog ugiba usled sile P = 20 kN

$$x = \sqrt{\frac{l^2 - a^2}{3}} = \sqrt{\frac{600^2 - 200^2}{3}} = 326 \text{ cm} \approx 300 \text{ cm} = \frac{l}{2}$$

$$f_{max}(q+P) = 1,1 + 0,72 = 1,82 \text{ cm} < 2,4 \text{ cm} = l/250 = f_{dop}$$

Analiza dobijenih rezultata



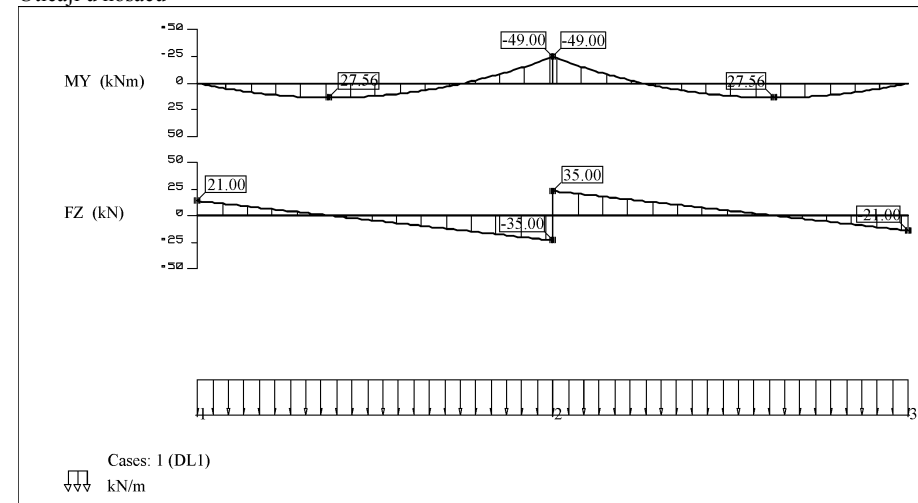
5b)

Osnovni materijal Č0361 (I sl.o.)  $\Rightarrow$  dopušten normalni napon  $s_{dop} = 16 \text{ kN/cm}^2$

dopušten smičući napon  $t_{dop} = 9 \text{ kN/cm}^2$

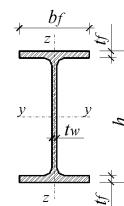
Dimenzionisanje nosača:

Uticaji u nosaču



$$W_{pot} = \frac{M_{max}}{s_{dop}} = \frac{4900}{16} = 306,25 \text{ cm}^3$$

Poprečni presek IPE 240 valjani profil



$b_f=120 \text{ mm}$	$A=39,1 \text{ cm}^2$	$W_y=324 \text{ cm}^3$
$t_f=9,8 \text{ mm}$	$I_y=3890 \text{ cm}^4$	$W_z=47,3 \text{ cm}^3$
$h=240 \text{ mm}$	$I_z=284 \text{ cm}^4$	$S_y=183 \text{ cm}^3$
$t_w=6,2 \text{ mm}$	$i_y=9,97 \text{ cm}$	$S_{y,o}=135,4 \text{ cm}^3$
	$i_z=2,69 \text{ cm}$	$I_z=12,9 \text{ cm}^4$

Kontrola napona:

Presek sa maksimalnim momentom u polju

$$s_{max} = \frac{M_{max}}{W_y} = \frac{2756}{324} = 8,5 \text{ kN/cm}^2 < s_{dop} = 16 \text{ kN/cm}^2$$

Presek nad srednjim osloncem

$$s_{max} = \frac{M_{max}}{W_y} = \frac{4900}{324} = 15,1 \text{ kN/cm}^2 < s_{dop} = 16 \text{ kN/cm}^2$$

$$t_{max} = \frac{T_{max} \cdot S_y}{t_w \cdot I_y} = \frac{35,0 \cdot 183}{0,62 \cdot 3890} = 2,65 \text{ kN/cm}^2 < t_{dop} = 9 \text{ kN/cm}^2$$

Kontrola uporednog napona

$$S = \frac{M_{\max}}{I_y} \cdot \frac{h - 2t_f}{2} = \frac{4900}{3890} \cdot \frac{(24 - 2 \cdot 0,98)}{2} = 13,9 \text{ kN/cm}^2 < S_{dop} = 16 \text{ kN/cm}^2$$

$$t = \frac{T \cdot S_{y,o}}{t_w \cdot I_y} = \frac{35 \cdot 135,4}{0,62 \cdot 3890} = 1,96 \text{ kN/cm}^2 < t_{dop} = 9 \text{ kN/cm}^2$$

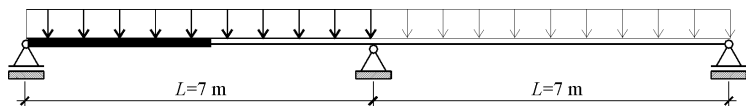
$$S_u = \sqrt{S^2 + 3 \cdot t^2} = \sqrt{13,9^2 + 3 \cdot 1,96^2} = 14,3 \text{ kN/cm}^2 < S_{dop} = 16 \text{ kN/cm}^2$$

Kontrola ugiba nosača \*

$$f = k \cdot \frac{q \cdot L^4}{I} = 0,248 \cdot \frac{8 \cdot 7^4}{3890} = 1,22 \text{ cm} < f_{dop} = L / 300 = 2 \text{ cm}$$

Kontrola stabilnosti nosača na bočno torziono izvijanje \*\*

1. Pritisnuta gornja nožica



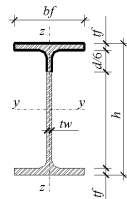
Razmak tačaka bočnog pridržavanja je:  $l_z = l_t = L / 2 = 3,5 \text{ m}$

Poluprečnik inercije dela preseka koga sačinjavaju pritisnuta nožica i 1/6 rebra:

$$A_f = b_f \cdot t_f = 12 \cdot 0,98 = 11,76 \text{ cm}^2$$

$$A_w = A - 2 \cdot A_f = 39,1 - 2 \cdot 11,76 = 15,58 \text{ cm}^2$$

$$i_{kz} = \frac{b_f}{\sqrt{12}} \cdot \sqrt{\frac{A_f}{A_f + A_w/6}} = \frac{12}{\sqrt{12}} \cdot \sqrt{\frac{11,76}{11,76 + 15,58/6}} = 3,13 \text{ cm}$$



Sen Venanov kritičan napon:

$$S_{Dv} = h_t \cdot \frac{P}{I_t \cdot W_y} \cdot \sqrt{EI_z \cdot GI_t} = h_t \cdot \frac{0,41 \cdot 10^5}{I_t \cdot W_z} \cdot \sqrt{I_z \cdot I_t} \text{ [kN/cm}^2\text{]}$$

$$S_{Dv} = 1,12 \cdot \frac{0,41 \cdot 10^5}{350 \cdot 324} \cdot \sqrt{284 \cdot 12,9} = 24,5 \text{ kN/cm}^2$$

Kritični napon deplanacije:

$$l_{kz} = \frac{l_z}{\sqrt{h_z \cdot i_{kz}}} = \frac{350}{\sqrt{1,12 \cdot 3,13}} = 105,7$$

$$S_{Dw} = p^2 \cdot \frac{E}{l_{kz}^2} = \frac{2,07 \cdot 10^5}{105,7^2} = 18,5 \text{ kN/cm}^2$$

Kritičan napon bočno-torzionog izvijanja

\* Čelične konstrukcije u građevinarstvu / str. 896

\*\* Čelične konstrukcije u građevinarstvu / JUS U. E7. 101 / str. 674

$$K = 1 + 0,156 \left( \frac{l_z}{h} \right)^2 \cdot \frac{I_t}{I_z} = 1 + 0,156 \left( \frac{350}{24} \right)^2 \cdot \frac{12,9}{284} = 2,51$$

$$f = \frac{\sqrt{K + r^2} - r}{\sqrt{K + r^2}} = \frac{\sqrt{2,51 + 0,46^2} - 0,46}{\sqrt{2,51 + 0,46^2}} = 0,72$$

$$S_{cr} = f \cdot \sqrt{S_{Dv}^2 + S_{Dw}^2} = 0,72 \cdot \sqrt{24,5^2 + 18,5^2} = 22,1 \text{ kN/cm}^2$$

Koeficijent oblika poprečnog preseka

$$a_p = \frac{W_{y,pl}}{W_{y,el}} = \frac{2 \cdot S_y}{W_y} = \frac{2 \cdot 183}{324} = 1,13$$

Relativna (bezdimenzionalna) vitkost

$$\bar{I}_D = \sqrt{\frac{a_p \cdot f_y}{S_{cr}}} = \sqrt{\frac{1,13 \cdot 24}{22,1}} = 1,108$$

Bezdimenzionalni koeficijent bočno torzionog izvijanja

$$c_D = \left( \frac{1}{1 + \bar{I}_D^{2n}} \right)^{1/n} = \left( \frac{1}{1 + 1,108^4} \right)^{1/2} = 0,632 \quad (n=2 \text{ za valjane profile})$$

Granični napon bočno torzionog izvijanja

$$S_D = a_p \cdot c_D \cdot f_y \leq f_y \quad S_D = 1,13 \cdot 0,632 \cdot 24 = 17,14 \text{ kN/cm}^2 < 24,00 = f_y$$

Kontrola napona

$$S = \frac{M}{W_y} = \frac{2756}{324} = 8,51 \text{ kN/cm}^2 < \frac{S_D}{n} = \frac{17,14}{1,5} = 11,43 \text{ kN/cm}^2$$

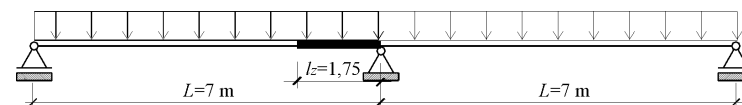
2. Pritisnuta donja nožica

Razmak tačaka bočnog pridržavanja je \*\*\*:

$$M(x) = \frac{3}{8} \cdot q \cdot l \cdot x - \frac{1}{2} \cdot q \cdot x^2 \quad \text{za } M(x)=0 \Rightarrow x = 5,25 \text{ m}$$

Dužina pritisnutog dela nožice je od nulte tačke dijagrama momenata do oslonca

$$l_z = 7 - 5,25 = 1,75 \text{ m} \quad l_t = 1,75 \text{ m}$$



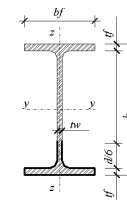
Sen Venanov kritičan napon:

$$S_{Dv} = h_t \cdot \frac{P}{I_t \cdot W_y} \cdot \sqrt{EI_z \cdot GI_t} = h_t \cdot \frac{0,41 \cdot 10^5}{I_t \cdot W_z} \cdot \sqrt{I_z \cdot I_t} \text{ [kN/cm}^2\text{]}$$

$$S_{Dv} = 1,77 \cdot \frac{0,41 \cdot 10^5}{175 \cdot 324} \cdot \sqrt{284 \cdot 12,9} = 77,5 \text{ kN/cm}^2$$

Kritični napon deplanacije:

$$l_{kz} = \frac{l_z}{\sqrt{h_z \cdot i_{kz}}} = \frac{175}{\sqrt{1,77 \cdot 3,13}} = 42,02$$



\*\*\* Metalne konstrukcije 1 / str. 520

$$s_{Dw} = p^2 \cdot \frac{E}{I_{kz}^2} = \frac{2,07 \cdot 10^5}{42,2^2} = 117,23 \text{ kN/cm}^2$$

Kritičan napon bočno-torzionog izvijanja

$$K = 1 + 0,156 \left( \frac{I_z}{h} \right)^2 \cdot \frac{I_t}{I_z} = 1 + 0,156 \left( \frac{175}{24} \right)^2 \cdot \frac{12,9}{284} = 1,377$$

$$f = \frac{\sqrt{K + r^2} - r}{\sqrt{K + r^2}} = \frac{\sqrt{1,377 + 0,46^2} - 0,46}{\sqrt{1,377 + 0,46^2}} = 0,635$$

$$s_{cr} = f \cdot \sqrt{s_{Dv}^2 + s_{Dw}^2} = 0,635 \cdot \sqrt{77,5^2 + 117,2^2} = 89,2 \text{ kN/cm}^2$$

Relativna (bezdimenzionalna) vitkost

$$\bar{I}_D = \sqrt{\frac{a_p \cdot f_y}{s_{cr}}} = \sqrt{\frac{1,13 \cdot 24}{89,2}} = 0,551$$

Bezdimenzionalni koeficijent bočno torzionog izvijanja

$$c_D = \left( \frac{1}{1 + \bar{I}_D^{2n}} \right)^{1/n} = \left( \frac{1}{1 + 0,551^4} \right)^{1/2} = 0,957 \quad (n=2 \text{ za valjane profile})$$

Granični napon bočno torzionog izvijanja

$$s_D = a_p \cdot c_D \cdot f_y \leq f_y \quad s_D = 1,13 \cdot 0,957 \cdot 24 = 25,95 \text{ kN/cm}^2 > 24,00 \Rightarrow s_D = 24 \text{ kN/cm}^2 = f_y$$

Kontrola napona

$s = \frac{M}{W_y} = \frac{4900}{324} = 15,12 \text{ kN/cm}^2 < \frac{s_D}{n} = \frac{24,00}{1,5} = 16,00 \text{ kN/cm}^2$
--

/ Usvaja se valjani profil IPE 240 /